Auctions

Auction = centralized protocol for redistributing resources among agents

Each agent attaches a value to each resource

Fixed (wrong) prices may inhibit deals:

- seller asks: value(seller) < value(buyer) < price
- buyer offers: price < value(seller) < value(buyer)

Auction = protocol to find price that allows deals

Applications of e-auctions

Stock and commodity markets
Bandwidth and electricity allocation
(band-x, Scandinavia)
Unique consumer goods (e-bay)
Load balancing, scheduling
Success criteria for auctions

Pareto efficiency =
resources end up with those who value them the most

Individual rationality =
Every participants gains non-negative utility

Auction settings

*Private value:* 
value of items depends only on agent’s preferences, e.g. haircut

*Common value:*
value of items determined entirely by other’s values, e.g. stocks, bonds

*Correlated value:*
value depends partly on own and other’s values, e.g. a Picasso painting
Classifying auctions

open-cry: bids are public vs.
sealed-bid: bids are secret

first-price: winner pays the highest bid vs.
second-price: winner pays the second-highest bid

<table>
<thead>
<tr>
<th></th>
<th>open-cry</th>
<th>sealed-bid</th>
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<tbody>
<tr>
<td>first-price</td>
<td>Dutch</td>
<td>Discriminatory</td>
</tr>
<tr>
<td>second-price</td>
<td>English</td>
<td>Vickrey</td>
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</table>

Auction protocols: Dutch

Auctioneer continuously lowers price until a bidder takes the item at the current price.

Strategy: down-bias bid depending on other bidders. Since it is unlikely that another bidder will have a valuation just below ours, we can bid a bit less and still probably win the auction.

Advantage: efficient, reveals only information about winner.

Dutch flower market, fish markets.
Auction protocols: English

Bidders raise their bids until nobody is willing to go any higher. The item is then sold to the highest bid.

Strategy: bid in small increments until own valuation is reached.

Advantage: no speculation (but long process)

Note: price paid = second highest + \( \epsilon \).

Art auctions, etc.

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Auction protocols: Discriminatory

Each bidder submits one secret bid, without knowing the others’ bids. The item is sold to the highest bidder.

Strategy: downward bias depending on other bidders

Advantage: one round of bidding, no information revealed.

Contracting in construction, etc.
Auction protocols: Vickrey

Each bidder submits one secret bid, without knowing the others’ bids. The item is sold to the highest bidder, but at the price of the 2nd-highest bid.

Strategy:
- bidding less: lower probability of winning the bid, but winning amount unchanged
- bidding more: same probability of profitable transaction, but possibility of unprofitable transaction

⇒ bid true valuation

Advantage: no speculation, single round of bidding, no information revealed.

Bidding strategies

*Truthful* bidding = bidding one’s true valuation

Optimal bidding: bid just enough to win the auction

⇒ take into account other bidders:
  - valuations
  - budgets
Optimal bids (risk-neutral, private value)

Assume values of bidders uniformly distributed

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<td>first-price</td>
<td>v(N-1)/N</td>
<td>v (N-1)/N</td>
</tr>
<tr>
<td>second-price</td>
<td>v</td>
<td>v</td>
</tr>
</tbody>
</table>

But with uniform distribution:
second-highest bid = highest bid * (N-1)/N
⇒ equivalent results (revenue equivalence theorem)

Human bidders

Risk-averse bidders:
Dutch, Discriminatory ≥ English, Vickrey
because speculation involves risk

People sometimes have irrational behavior:
Dutch auction: bid lower because of suspense
Vickrey auction: bid higher because you won’t pay that price
Non-private value settings

Other bids influence own bid
Revenue non-equivalence:
English $\geq$ Vickrey $\geq$ Dutch = first-price sealed bid

Vulnerability to collusion

Collusion = group of buyers coordinate their bidding
Suppose $v_1 = 20$, $v_i = 18$, $i \geq 2$

English auction: Agent 1 bids 6, all others bid 5; self-enforcing
Vickrey: Agent 1 bids 20, all others bid 5; self-enforcing
Dutch/Discriminatory: If Agent 1 bids below 18, others are motivated to break the agreement
Bidding strategies

Auction = competition with other buyers
⇒ strategy depends on model of others:
  • valuations ⇒ how much is needed to win
  • budgets ⇒ how much can they pay
  • risk profiles

Multiple units

Example:
  • 4 identical items
  • 4 buyers willing to pay Fr. 400, 300, 200, 100
⇒ selling prices:
  • first item sold for Fr. 400
  • second item sold for Fr. 300
  • third item for Fr. 200
  • fourth item for Fr. 100
Problems with multiple items

Unfair! Everybody pays a different price.

Susceptible to manipulation, e.g. demand reduction lie:

- 4 items for sale
- agent 1: valuation for first item = Fr. 400,
  for second item = Fr. 200
- other agents want to buy 3 items at Fr. 500, 300, 150
  ⇒ price for getting one item ≃ Fr. 0
  ⇒ price for getting two items ≥ 2 * Fr. 150 = Fr. 300

Utility for buying one = Fr. 400, for both = Fr. 300 only.

Generalizations

Uniform price auction:

  n units for sale ⇒ each agent pays n + 1st highest bid

still subject to demand reduction lie

Multi-unit Vickrey auction:

  Each agent pays price of the bid it displaced from the set of
  winning bids

  ⇒ truthful bidding the best strategy
Example (multi-unit Vickrey)

4 items
5 buyers, valuations = Fr. 500, 400, 300, 200, 150
⇒ Fr. 150 bid is displaced
⇒ everyone pays Fr. 150
Note: significantly lower revenue!

<table>
<thead>
<tr>
<th>Combination</th>
<th>Val. (agent1)</th>
<th>Val. (agent2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>{a}</td>
<td>1</td>
<td>1.5</td>
</tr>
<tr>
<td>{b}</td>
<td>2</td>
<td>1.5</td>
</tr>
<tr>
<td>{c}</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>{a, b}</td>
<td>4</td>
<td>3.5</td>
</tr>
<tr>
<td>{a, c}</td>
<td>3</td>
<td>3.5</td>
</tr>
<tr>
<td>{b, c}</td>
<td>3</td>
<td>2.5</td>
</tr>
<tr>
<td>{a, b, c}</td>
<td>6</td>
<td>5.5</td>
</tr>
</tbody>
</table>

Multiple items

Buyers are looking for *combinations* of items

Interrelated auctions: bidding takes place in sequence
Combinatorial auctions: bidding takes place in parallel
Interrelated auctions

Bidding in sequence:

1. item a:
   both agents bid for \( \{a\} \)
   agent 2 wins with 1.5

2. item b:
   agent 1 bids for \( \{b\} \), agent 2 bids for \( \{a, b\} - \{a\} \)
   agent 1 wins with 2

3. item c:
   agent 1 bids for \( \{b, c\} - \{b\} \), agent 2 bids for \( \{a, c\} - \{a\} \)
   agent 2 wins with 2

Total payoff for auctioneer: 5.5 units

Weakness of interrelated auctions

Uncertainty about future auctions requires speculation.

Example: items \( \{L, R\} = \) left shoe, right shoe

Only the pair has a value ⇒

- bid low in first auction to compensate for risk of not getting the second item.
- bid low in the second auction because only the one that got the first item bids.

⇒ combinatorial auctions
Combinatorial auctions

Combinatorial auction:
- bidders place bids for combinations of items
- auctioneer decides on best combination of bids

Number of bids exponential in number of items
Number of deals exponential in number of bids
⇒ determining the winning combination is NP-hard!

Feasible combinations

Feasible combinations = combinations where each item is sold only once:

Generalized Vickrey Auction (GVA): Agent pays for difference between best allocation without its bids and best allocation with its bids: Agent 1 buys all items and pays 5.5.
Problems with GVA

GVA can be manipulated. Example: false-name bids

Agents $A_1$, $A_2$ are interested in goods $a$ and $b$ with valuations:

<table>
<thead>
<tr>
<th></th>
<th>$a$</th>
<th>$b$</th>
<th>$a+b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>$x$</td>
<td>$x$</td>
<td>$2x$</td>
</tr>
<tr>
<td>$A_2$</td>
<td>$0$</td>
<td>$0$</td>
<td>$x+y$</td>
</tr>
</tbody>
</table>

Assume $x > y \Rightarrow A_1$ gets $a+b$ and pays $x+y$.

But by introducing a fake bidder, $A_1$ can get the same goods at price $2y$!

GVA with false-name bid...

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<td>$A_2$</td>
<td>$0$</td>
<td>$0$</td>
<td>$x+y$</td>
</tr>
<tr>
<td>$A'_1$</td>
<td>$0$</td>
<td>$x$</td>
<td>$x$</td>
</tr>
</tbody>
</table>

Assume $x > y \Rightarrow$ best solution (pareto-efficient):

$A_1$ gets $a$, $A'_1$ gets $b$, each pays $y$

$\Rightarrow A_1$ gets both goods at price $2y < x + y$

No truth-incentive, pareto-optimal mechanism can avoid this problem!
Double auctions

\[ \begin{array}{c}
\times = \text{sell}(M \text{ offers}) \\
\circ = \text{buy} \\
(M+1)\text{st} \quad M-\text{th} \\
\text{bid} \quad \text{ask} \\
\end{array} \]

M sell and N buy bids ⇒ price determination:

- M-th highest price (ask quote): incentive-compatible for sellers
- (M+1)-st highest price (bid quote): incentive-compatible for buyers
- fraction in between

Nothing is incentive-compatible for both sellers and buyers!

Clearing the market

All buy bids \( \geq \) M-th highest have a sell bid \( \leq \) M-th highest
⇒ can always match buyers and sellers at price \( p \),

\( (M+1)\text{-st} \leq p \leq M\text{-th} \)

⇒ new price quotes; no further transactions
Auction platforms

- Provide information exchange
- Implement auction protocol
- Determine winner and conditions
- Possibly: Guarantee protocol/authenticity

Issues in information exchange

- Modelling goods
- Publishing auctions
- Registering buyers
Modelling goods

Cars: model, age, mileage, options
Oil: amount, type, delivery date and place
Network bandwidth: where, when, what amount
Buyers always assume the worst:
missing information = negative information
⇒ information should be complete

Publishing auctions
Registering bidders

Should only consider serious bids:

- obligation to buy if winner
- no phantom bids to perturb the market

⇒ important to register buyers

e-bay: verification of e-mail address
band-x: open an account, be a telecom company
stock market: buy a license (in an auction!)

Implementing auction protocols

Publishing auction state
Receiving, authenticating and sequencing bids
Using local agents
Publishing auction state

Open-cry auctions: continuously changing state

Challenge: publish state so that all bidders have the same information at the same time

Requires very complex technology (Swiss Stock Market!)

Impossible to guarantee over the internet

Receiving and processing bids

Bids must be authentic:

⇒ verification using cryptography

Bids must be confidential:

⇒ requires encryption

Contradictory requirements (see cryptography course)

⇒ problematic for sealed-bid/Vickrey auctions
Timing issues

Bid timing can be crucial:

- Dutch:
  - delayed bid may loose
  - fake delayed bid may win
- Vickrey/sealed-bid: bid could be inserted after closing and winner announcement

Internet latency ⇒ tradeoff:

\[ \text{speed of auction} \Leftrightarrow \text{security} \]

Agents as a way out of the dilemma

Timing impossible to guarantee through the internet

Run \textit{bidder agents} locally on the server:

- control over each agent’s information state
- bid timing is known exactly

Problems:

- \textit{embeddedness}: cannot tolerate infinite loops
- \textit{confidentiality}: can decode bidder’s strategy

⇒ require new solutions
Interagents

Registered bidders can be identified by *interagents*:

- cryptographic protocol between interagent and bidder
- interagent can handle timing functions

Agent’s Logic:
Inner and Social Behavior

Interaction Issues
Communication and Conversation Support

Fishmarket

Double auctions

New buyers and sellers appear continuously

Quotes =
- *bid* quote = max. price to sell an item
- *ask* quote = min. price to buy an item

given the current set of bids

bid \leq ask \Rightarrow matching bids

Can only clear at certain time intervals

⇒ ask quote can be higher than bid quote; this is correct information
Winner determination

Single item: according to protocol
Multi-item/divisible items
Combinatorial auctions

Divisible items

Divisible goods: multiple buyers/sellers
Uniform price: everyone should pay the same!
(avoiders of counterspeculation)
Periodic clear ($M = \text{number of sellers}$)
  - $M$-th price (neutral to sellers)
  - $(M+1)$-st price (neutral to buyers)
Continuous clear: immediately satisfy any possible match.
Example (divisible items)

<p>| | | | |</p>
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<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Agent A</td>
<td>sell 1 unit at Fr. 2</td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>Agent B</td>
<td>buy 1 unit at Fr. 3</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>Agent C</td>
<td>buy 1 unit at Fr. 4</td>
<td></td>
</tr>
</tbody>
</table>

M-th price: A sells to C at Fr. 4
C could bid $3 + \epsilon$

(M+1)-st price: A sells to C at Fr. 3
A should ask for $4 - \epsilon$

Continuous: A sells to B at Fr. 2
Suboptimal sale!

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- Used in bandwidth/electricity auctions
- Auction result = split up offer to maximize revenue
- Everybody pays the same: sum up curves and demands
- Bids may be price-quantity graphs

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Quantity
Price
Winners in combinatorial auctions

Combinatorial auction: bids are for *combinations* of goods
Winners = set which maximizes total revenue
Very complex problem, impossible to approximate
Tractable search when space of bids only sparsely populated

After winner determination

Buyer/seller need to exchange money/goods
Large room for fraud
Auction platform can provide trusted contracts (e.g. Christie’s, Sotheby’s)
Summary

Auctions = mechanism for reallocating resources

Goal of auction protocols:
- goods end up with those who value them the most
- encourage truthful bidding

Goal of participants: maximize benefit

Irrationality ⇐ risk-aversion

Auction platforms for multi-agent auctions