Achieving Budget-Balance with Vickrey-Based Payment Schemes in Exchanges


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Outline

☑ Problem & Approach
☑ Vickrey Payments
☑ Budget-Balance in Vickrey Scheme
☑ Budget-Balance Payment rules
☑ Theoretical Analysis
☑ Experimental Analysis
☑ Discussion & Conclusions

Combinatorial exchanges

Problem

☑ Combinatorial exchange problems:
  • Efficient?
  • Budget-balanced?
  • Individual-rationality (IR)?
  • Incentive-compatibility (IC) (Strategy-proofness)?

☑ Generalized Vickrey mechanisms:
  • Efficient
  • Strategy-proofness
  • But not Budget-balanced!

☑ Impossible result (Myerson & Satterthwaite, 1983):
  • No exchange can be efficient, budget-balanced, and individual-rational.

Approach

☑ Goals:
  • Allocative-efficiency
  • Budget-balance (BB)
  • Individual-rationality (IR)
  • Incentive-compatibility (IC)

☑ Approaches:
  • Impose BB and IR, fairly efficient but IC
  • Impose BB and IR, fairly efficient and fairly IC

Authors’ approach: Vickrey-based scheme

• Adapting the Vickrey payment scheme to make it BB

Vickrey Payments

• The payment of an agent is the net increment in total surplus that his participation creates.

• Vickrey payment for an agent i:
  \[ p_{Vick,i} = (V_i) - V_i^* \]

• Vickrey discount for an agent i:
  \[ \Delta_{Vick,i} = V_i^* - (V_i) \]

\( V_i^* \): reported value of agent i
Budget-Balance in Vickrey Scheme

- Budget-Balance fails with Vickrey payments in exchanges in general!

Example: Suppose agents $\{1,2,3,4\}$ and items $\{A,B\}$.

Agent 1: sells A with $v_1(A) = 10$
Agent 2: sells B with $v_2(B) = 5$
Agent 3: buys AB with $v_3(AB) = 51$
Agent 4: buys AB with $v_4(AB) = 40$

$V^* = 51\cdot10 - 5\cdot36 = 36$. $V^* - 1^* = (V^* - 2^*) = 0$, $V^* - 3^* = 25$, $V^* - 4^* = 36$

$p_{Vick,1} = -10 - (36 - 0) = -46$, $p_{Vick,2} = -5 - (36 - 0) = -41$, $p_{Vick,3} = 51 - (36 - 25) = 40$.

$\Rightarrow$ The exchange runs at a loss of $\$47$ to the market maker.

Budget-Balance in Vickrey Scheme (cont.)

- Some special cases with Budget-Balance:
  - One-side Vickrey payments: If Vickrey payments are implemented on one-side of an exchange, and no aggregation in that side.
  - One-to-N models: BB fails with Vickrey payments to all agents in a combinatorial auction except in the case that no buyer requires a Vickrey discount.

Budget-Balance Payment rules

- Goal: Minimizing the distance to Vickrey payments $L(A, A_{Vick})$ with BB and IR as hard constraints.

- Possible distance functions:
  - $L_2$, $L_¥$
  - $L_{RE}(A, A_{Vick}) = \sum |A_i - A_{Vick}i|$ (relative error function)
  - $L_{rel} = \sum |A_i - A_{Vick}i|$ (squared relative error function)
  - $L_{wei} = \sum A_i - A_{Vick}i$ (weighted error function)
  - $L_p = \prod |A_i - A_{Vick}i|$ (product error function)

Distance Functions and Payment Schemes

- Calculated using Lagrangian optimization:

<table>
<thead>
<tr>
<th>Distance Function</th>
<th>Payment Scheme</th>
<th>Discount Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_2$, $L_¥$</td>
<td>Threshold</td>
<td>$\max(0, \Delta - C)$</td>
</tr>
</tbody>
</table>
| $L_{RE}$          | Fractional     | $\Delta_{Vick}$ if $\Delta_{Vick} \leq C$
| $L_{WE}$          | Large          | $\min(\Delta_{Vick}, C)$ |
| $L_p$             | Reverse        | $\Delta = \frac{\Delta_{Vick}}{C}$ |

Theoretical Analysis

- Exchange with single item
  - Each agent $i$:
    - Its value $v_i$ manipulated by $b_i$, and bid $b_i = v_i - \delta_i$
    - The maximum bid (ask) $x_i$ from others agents is uniformly distributed about $v_i$, i.e., $x_i \sim U(v_i - \delta, v_i + \delta)$ with $\delta > 0$
    - The average surplus available: $\alpha \delta$ per-agent, with the amount of surplus $\alpha > 0$
  - Payment scheme parameters: $C, \mu, D$. 

Payment $b_i - \Delta_i$ vs. Bid price $b_i$

- equal
- reverse
- larger
- smaller
- threshold
- strict
Utility functions vs. highest outside bid x

Utility functions for different settings:
- (a) Vickery
- (b) No-Discount
- (c) Threshold
- (d) Fractional

Analytical results

- Calculated using the assumption of uniform distribution of x_i around x_k:

<table>
<thead>
<tr>
<th>Rule</th>
<th>Optimal Manipulation, θ^ *</th>
<th>Expected Discount</th>
</tr>
</thead>
<tbody>
<tr>
<td>No-Discount</td>
<td>d/2</td>
<td>δ/4</td>
</tr>
<tr>
<td>Vickery</td>
<td>0</td>
<td>χ</td>
</tr>
<tr>
<td>Fractional</td>
<td>max[0, (C/2d)]</td>
<td>min[δ/4, (C/2d)]</td>
</tr>
<tr>
<td>Threshold</td>
<td>min[C_d/2]</td>
<td>max[0, (d - 2C)θ]</td>
</tr>
<tr>
<td>Equal</td>
<td>δ = D</td>
<td>δ(1 + δ)</td>
</tr>
<tr>
<td>Small</td>
<td>max[0, min[δ/2, δ - C]]</td>
<td>min[δ/4, C/4d]</td>
</tr>
<tr>
<td>Large</td>
<td>0, if C ≤ δ/2</td>
<td>-C^2/4δ + δ/4, if C ≤ 4d/3</td>
</tr>
<tr>
<td>Reverse</td>
<td>max[0, 4dC]</td>
<td>min[δ/4, C/4d]</td>
</tr>
</tbody>
</table>

Expected gain in utility vs. manipulation θ

Expected gain in utility Eu(θ) - Eu(0), with amount of surplus α=0.1

Optimal agent manipulation

Agent manipulation vs. α when the amount of surplus α increases from 0 to d/2 per-agent

Experimental Analysis

- Assumptions:
  - Strategy of agent l: adjust all its bids and asks by the same fractional amount y
  - Look for a symmetric Nash equilibrium for some y
  - Compute an approximation to the equilibrium

- Combinatorial Exchanges:
  - 5, 10, 20 agents
  - 100 bids and asks
  - 50 goods

Average Single-Agent Gain in Utility

Problem size: 5 buyers/5 sellers
Experimental results
Computed at manipulation levels: 10%, 20%, 30%

<table>
<thead>
<tr>
<th></th>
<th>No-Discount</th>
<th>Vickrey</th>
<th>Small</th>
<th>Frac</th>
</tr>
</thead>
<tbody>
<tr>
<td>Utility Gain</td>
<td>0.799</td>
<td>-0.195</td>
<td>0.479</td>
<td>0.590</td>
</tr>
<tr>
<td>Correlation</td>
<td>0.053</td>
<td>1.0</td>
<td>0.356</td>
<td>0.590</td>
</tr>
<tr>
<td>Manipulation $\theta^*$</td>
<td>48</td>
<td>0</td>
<td>48</td>
<td>32</td>
</tr>
<tr>
<td>Efficiency (%)</td>
<td>58</td>
<td>100</td>
<td>58</td>
<td>78</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Threshold</th>
<th>Equal</th>
<th>Large</th>
<th>Reverse</th>
</tr>
</thead>
<tbody>
<tr>
<td>Utility Gain</td>
<td>0.110</td>
<td>0.516</td>
<td>0.029</td>
<td>0.337</td>
</tr>
<tr>
<td>Correlation</td>
<td>0.543</td>
<td>0.356</td>
<td>0.176</td>
<td>0.522</td>
</tr>
<tr>
<td>Manipulation $\theta^*$</td>
<td>42</td>
<td>46</td>
<td>18</td>
<td>44</td>
</tr>
<tr>
<td>Efficiency (%)</td>
<td>80</td>
<td>62</td>
<td>88</td>
<td>64</td>
</tr>
</tbody>
</table>

Discussions
- The ordering from the experimental results is consistent with the results of the theoretical analysis:
  \((\text{Large, Threshold}) > \text{Fractional} > \text{Reverse} > (\text{Equal, Small})\)
- Allocative efficiency in the Large and Threshold.
- Large is riskier than Threshold.

Conclusions
- Vickrey payments are not budget-balanced!
- Budget-balanced payment schemes can be constructed by minimizing different distance functions to Vickrey payments.
- A simple Threshold rule has better incentive properties than other payment schemes.

Thank you!
Questions?